Int. J. Heat Mass Transfer. Vol. 21, pp. 145–154. Pergamon Press 1978. Printed in Great Britain

INFLUENCE OF END CONDUCTION ON THE SENSITIVITY TO STREAM TEMPERATURE FLUCTUATIONS OF A HOT-WIRE ANEMOMETER

K. BREMHORST* and D. B. GILMORE†
Department of Mechanical Engineering, University of Queensland,
St. Lucia, Q4067, Australia

(*Received* 29 *November* 1976 *and in revised form* 9 *June* 1977)

Abstract—The influence of end conduction on the sensitivity to stream temperature fluctuations of a constant current hot-wire anemometer operated at low overheat has been investigated using a newly developed dynamic, cross-correlation calibration technique. A lumped parameter heat-transfer model of the wire and support system consistent with the experimental data is presented. Results show that for commonly used tungsten and platinum wires, a frequency dependent sensitivity to stream temperature fluctuations is obtained in air flows at low subsonic velocities. For measurements with an accuracy of approximately 5% under non-isothermal flow conditions, use of a wire of low thermal conductivity such as Pt 10% Rh with a length-to-diameter ratio of at least 400, irrespective of wire diameter, is recommended compared with 800 for equivalent accuracy with tungsten wire. The relevance of the results to velocity measurements with constant temperature hot-wire anemometers in non-isothermal flows is also discussed.

NOMENCLATURE

A, voltage-displacement sensitivity of displacement transducer;
$B', B'', \ldots$, first, second, etc., derivatives in Taylor series expansion of δ as a function of y;
$C_{1,2,3}$, thermal capacitance associated with the hot-wire, transition zone and supports respectively;
d, wire diameter;
e, output voltage change;
E, total output voltage;
h, convection heat-transfer coefficient;
$\bar{I}$, mean wire current;
j, $=\sqrt{-1}$;
k, thermal conductivity;
L, wire length;
q, heat input to hot-wire due to Joule heating;
R, ratio of resistances defined by equation (4b);
$R\ldots$, electrical or thermal resistance;
R', equivalent resistances defined by Fig. 3(b);
R'', ratio of resistances defined by equation (4c);
s, $=j\omega$;
t, time;
T, instantaneous temperature = time average + fluctuation;
$\tilde{T}_1$, wire temperature fluctuation;
$T_1(s)$, wire temperature fluctuation at frequency ω;
U, velocity;
y, positional co-ordinate normal to flow direction.

Superscript

——, time average.

Subscripts

∞, infinitely long wire: wire with negligible end losses;
c, conduction of heat between subsystems;
DT, displacement transducer;
E, equilibrium temperature; equals stream temperature at low subsonic speeds;
h, convection heat transfer;
k, conduction heat transfer to a constant temperature sink;
m, measured quantity with finite length wire: one with significant end conduction;
n, noise;
0, reference state;
S, support which includes any end plating used on the wire;
$+y, -y$, positive and negative displacements normal to the flow direction from a reference position;
1, 2, 3, subsystems—wire, transition zone and support respectively.

Greek symbols

α, thermal coefficient of resistance;
δ, stream temperature fluctuation;
Δ, change in variable from one steady state position to another;
τ_1, τ_2, time constants, τ_2 being that of the support and τ_1 that where influence of support on dynamic response of system becomes frequency independent for increasing frequencies;
ϕ, phase angle;
ω, angular frequency.

Other symbols

$||$, absolute value.

* Senior Lecturer.
† Postgraduate Student.

INTRODUCTION

THE HOT-WIRE anemometer is a frequently used instrument for the measurement of fluid temperature fluctuations in non-isothermal flows. For such measurements it is generally operated at low wire currents so that Joule heating of the wire is negligible and the wire responds like a resistance thermometer. When the wire is effectively long, thermal losses to the wire end supports are negligible compared with the convected heat losses. The wire temperature is then directly related to the stream temperature—the wire behaving like a single time constant system for which stream and wire temperatures are equal for stream temperature fluctuation frequencies well below the wire's corner frequency. Above the wire's corner frequency, the wire temperature decreases with increasing frequency due to thermal inertia effects.

When a constant current, $\bar{I}$, is passed through such a wire, the output voltage change for a given change in stream temperature, δ, at sufficiently low frequency, is given by

$$e_r = \bar{I} R_0 \alpha_0 \tilde{T}_1 \quad \text{where} \quad \tilde{T}_1 = \delta. \tag{1}$$

It will be assumed throughout that the spatial scale of δ is sufficiently large so that δ is invariant with position along the wire and its supports. Radiant heat-transfer and thermoelectric effects will be neglected.

Frequently, effectively short wires are used for which thermal interaction between the wire and its end supports may not be negligible. Maye [1] analysed this situation for the case of the wire and its end supports being at different temperatures under essentially steady state conditions. Because of conduction of heat to or from the supports, any wire temperature change will not equal the stream temperature change, that is, $\tilde{T}_1$ differs from δ. Unfortunately, Maye [1] presented neither a reliable calibration technique nor accurate experimental verification of the theoretically developed results. Furthermore, by neglecting the thermal inertias of the wire and its supports, the frequency dependence of this end conduction on the wire's transfer function has been suppressed.

Højstrup *et al.* [2] extended the analysis by solving the differential equation giving the instantaneous wire heat balance. The analysis includes the effect of the wire conduction losses to the supports which are assumed to respond to ambient temperature fluctuations like a first-order low-pass filter with their own time constant. The mean temperature of the supports was, however, assumed to equal the mean ambient temperature. Verification of the theory was attempted with measurements of sound generated temperature fluctuations in a cavity. No results for non-isothermal fluid flow are presented. Also, from the practical viewpoint, little direct indication is given of what type of wire would give a frequency independent transfer function, at least within normal experimental accuracies.

Considering the wire supports as a simple pin fin and applying the standard solutions for the temperature distribution along such a fin (for example, Holman [3]) shows that the fin is of high efficiency for typically used dimensions and materials. Thus, if the end of the support embedded in the probe stem is at a different mean temperature to that of the flow at the point of measurement—a situation to be expected in the presence of mean flow temperature gradients—the mean temperature at the wire ends will not equal the mean flow temperature. The latter situation is not catered for by Højstrup *et al.* [2].

This paper studies the role of end conduction heat losses and extends the lumped parameter model of the wire and support system used by Bremhorst *et al.* [4] to the case of stream temperature fluctuation measurements. The conditions under which wire temperature perturbations produced by wire current pulsations give an identical transfer function to that produced by stream temperature fluctuations are identified. This latter aspect is of importance when using the current pulsation technique for the matching of an open-loop compensation network to the wire's response in order to extend the useful frequency range of the system. Finally, the relevance to velocity fluctuation measurements made with hot-wire anemometers in non-isothermal flows is examined.

MODEL OF HOT-WIRE AND SUPPORT SYSTEM

Figure 1 shows the thermal model of the hot-wire and its supports recently proposed by Bremhorst *et al.*

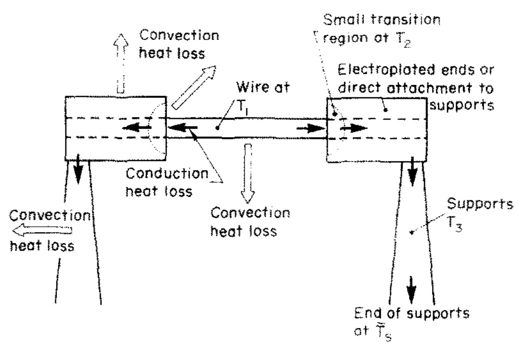

FIG. 1. Physical model of hot-wire.

[4] who showed from current perturbation measurements that the wire-support system consists of three readily identifiable sub-systems: the wire, a small transition region at the ends of the wire which has a time constant slightly less than that of the wire, and the bulk of the end support. A lumped parameter equivalent network for the case of fluctuating stream temperature with the free end of the wire supports being at some temperature, $\bar{T}_S$, other than $\bar{T}_E$ is shown in Fig. 2. Since only the response under conditions of negligible Joule heating is of interest, that is, operation as a resistance thermometer, $q \simeq 0$. At perturbation frequencies well below the wire corner frequency but still of the order of the corner frequency of the end supports, the simplified model of Fig. 3(a) results,

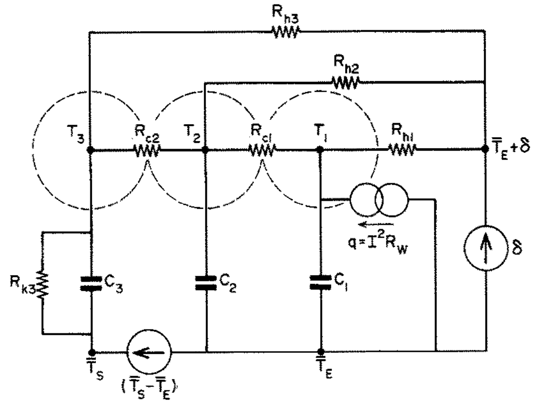

FIG. 2. Lumped parameter network equivalent circuit of hot-wire for fluctuating $\bar{T}_E$. $q \simeq 0$ when Joule heating is negligible.

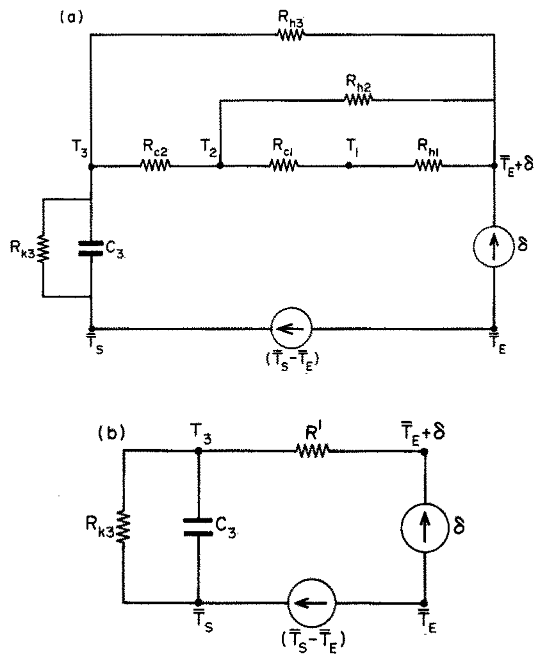

FIG. 3(a). Simplified equivalent circuit for perturbation frequencies below the wire corner frequency and negligible Joule heating. (b) Regrouping of circuit of Fig. 3(a).

whereas at frequencies well above that of the end support's corner frequency, the simplified circuit of Fig. 4(a) applies. It is assumed that the corner frequencies of the wire and the bulk of the end support are well separated.

The circuit of Fig. 3(a) can be further simplified to that of Fig. 3(b) where R' is the equivalent of all resistances between nodes T_3 and $\bar{T}_E+\delta$. The measured change in output from the hot-wire consequent upon the introduction of a stream temperature change, δ, is given by equation (2).

$$e_m = \bar{I}R_0\alpha_0\,\dot{T}_1 \tag{2}$$

whereas that for a wire with no end losses, the ideal or infinite wire, is given by equation (1). The ratio e_m/e_∞ of equation (3a) or that of equation (3b) when working in the frequency domain is required for an assessment of the effect of end conduction.

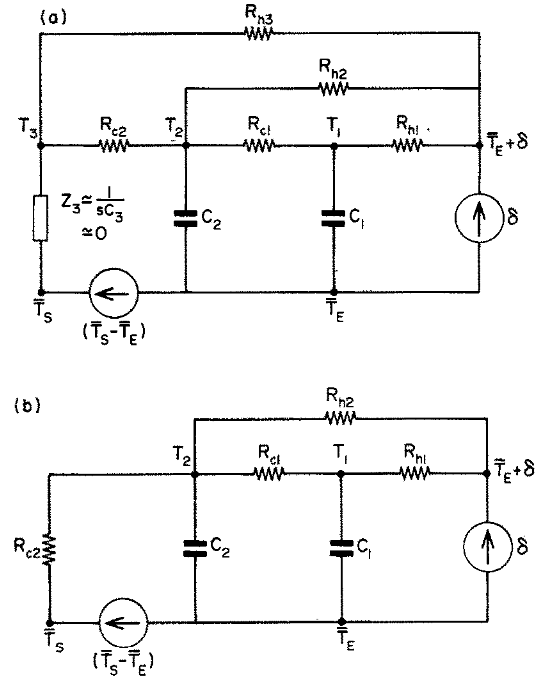

FIG. 4(a). Simplified equivalent circuit for perturbation frequencies well above the corner frequency of the end supports and negligible Joule heating. (b) Simplification of Fig. 4(a).

$$e_m/e_\infty = \dot{T}_1/\delta \tag{3a}$$

$$e_m(s)/e_\infty(s) = T_1(s)/\delta(s). \tag{3b}$$

At frequencies below the wire corner frequency, it can readily be shown in terms of the elements of the circuits of Figs. 3(a) and (b) that the perturbation response of the system for variations about $\bar{T}_E$ is given by equation (4a)

$$\frac{T_1(s)}{\delta(s)} = [1-R''R]\left[\frac{1+\tau_1 s}{1+\tau_2 s}\right] \tag{4a}$$

where

$$R = \frac{R'}{R'+R_{k3}} \tag{4b}$$

$$R'' = \frac{R_{h1}}{R_{h1}+R_{c1}+\dfrac{R_{c2}}{R_{h2}}(R_{c1}+R_{h1}+R_{h2})} \tag{4c}$$

$$\simeq \frac{1}{1+R_{c1}/R_{h1}} \quad \text{for large } R_{h2}$$

$$\tau_2 = C_3 R R_{k3} \tag{4d}$$

$$\frac{\tau_1}{\tau_2} = \frac{1-R''}{1-R''R}. \tag{4e}$$

It is noteworthy that the perturbation response is the same for $\bar{T}_S = \bar{T}_E$ as for $\bar{T}_S \neq \bar{T}_E$.

The model investigated by Højstrup *et al.* [2] corresponds to the case of $R_{k3} = \infty$ with $\bar{T}_S = \bar{T}_E$,

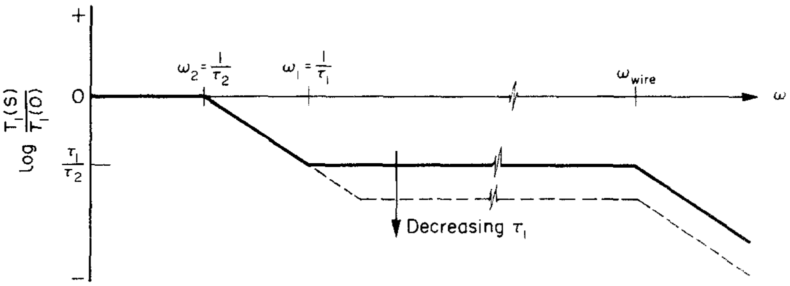

FIG. 5. Asymptotic form of equation (7)—amplitude only—with first order system approximation for wire and transition zone at higher frequencies.

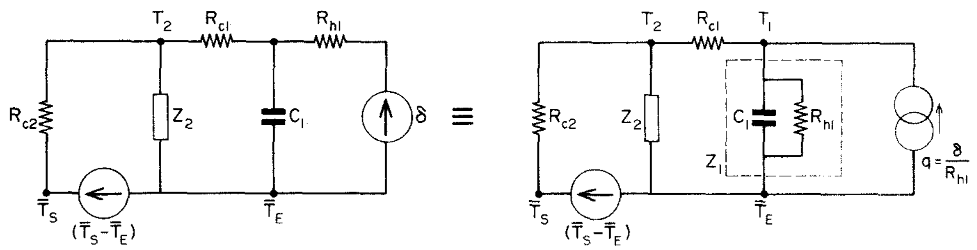

FIG. 6. Simplified equivalent circuit for perturbation frequencies well above the corner frequency of the supports for $R_{h2} \to \infty$ and negligible Joule heating.

reducing equation (4a) to equation (5).

$$\frac{T_1(s)}{\delta(s)} = \frac{1+\tau_1 s}{1+\tau_2 s} \tag{5}$$

where $\tau_1/\tau_2 = 1 - R''$ and $\tau_2 = C_3 R'$.

This latter model assumes that the time constant of the end support, τ_2, is determined by the resistance to convective heat transfer to or from the supports, particularly, when R_{c1} and R_{c2} are relatively large (R_{h2} can generally be expected to be very large). Equation (4d) shows however, that when $R_{k3} < R'$, τ_2 will approach $C_3 R_{k3}$ which will lead to a smaller time constant for the support than if convection heat transfer alone exists.

The accurate measurement of $\delta(s)$ for substitution into equation (4a) is extremely difficult because even when precautions against end conduction losses are taken, an uncertainty will always exist. Use of a thermocouple for this measurement does not improve the situation when operating in a flow with a mean temperature gradient. $T_1(s)$ at $s = 0$ is, however, a very simple and accurate measurement which when performed during a dynamic calibration requires no separate or new probe setting-up within the flow. This zero frequency value of T_1, $T_1(0)$, is given by equation (6) from equation (4a) and can be used to normalize the latter to the form of equation (7), where a constant size perturbation in stream temperature has been assumed, that is, $\delta(0) = \delta(s)$.

$$\frac{T_1(0)}{\delta(0)} = (1 - R''R) \tag{6}$$

$$\frac{T_1(s)}{T_1(0)} = \frac{1+\tau_1 s}{1+\tau_2 s}. \tag{7}$$

The amplitude variation of this function with frequency is shown in Fig. 5 in asymptotic form for two values of τ_1/τ_2 and fixed τ_2.

At higher frequencies—of the order of the wire corner frequency—the equivalent circuit of Fig. 4(b) applies where R_{h3} is not shown as the loop containing it becomes independent of the remaining circuit. If R_{h2} is very large, which is probably a good approximation in the absence of more substantial data than are available at present, the circuit of Fig. 6 results. After substitution for the voltage source, δ, with the current source equivalent, q, it is seen that stream temperature fluctuations and wire heat input perturbations produce an identical response within the thermal network representing the wire and its supports. Wire heat input perturbations can be easily produced by the long established method of perturbing the wire heating current thus providing a simple means of measuring this section of the wire response. Application of this latter technique to the case of $\bar{T}_S = \bar{T}_E$ has shown that for wires with end conduction losses, a precisely first order system response at these frequencies is not obtained (Bremhorst *et al.* [4]) although for purposes of illustration, such an approximation is used in Fig. 5.

The measurement of the wire response over the whole frequency range generally of interest, by means of a single method, represents an as yet unanswered challenge. Since Bremhorst *et al.* [4] have already measured the response at higher frequencies for the complete system, only the lower frequency range requires further consideration. A method of obtaining this by shaking the wire in a temperature gradient to produce stream temperature perturbations is described in the following sections.

DYNAMIC CALIBRATION TECHNIQUE—THEORY

Movement of a hot-wire through a known linear temperature gradient in a uniform velocity field, will produce a known temperature perturbation. The production of such a flow field at normally used flow Reynolds numbers without associated turbulent temperature fluctuations is nearly impossible. The anemometer output will, therefore, contain the oscillating component produced by the movement of the hot-wire as well as an uncorrelated turbulent temperature fluctuation component. In addition, unless the wire is completely insensitive to velocity fluctuations, a contribution from these will also be present and since temperature sensitive wires are operated at very low wire currents, electronic noise contributions can also be expected. Like the turbulent temperature fluctuations, turbulent velocity fluctuations and electronic noise will be uncorrelated with the perturbation component—since the velocity field is uniform, only a velocity perturbation normal to the flow direction and 90° out of phase and hence uncorrelated with the temperature perturbation will result.

If a signal proportional to the temperature perturbation is available simultaneously with the output signal from the hot-wire, the cross-correlation technique reported recently by Bremhorst and Gilmore [5] for the determination of the hot-wire's sensitivity to velocity fluctuations in the presence of uncorrelated extraneous effects, can be applied to obtain a dynamically determined sensitivity to temperature perturbations. When the wire is moved through a linear temperature gradient, a signal proportional to the temperature perturbation can readily be obtained by use of a displacement transducer attached to the probe supporting the wire. This can be correlated with the hot-wire signal. The noise free hot-wire output is given by equation (2) whereas the displacement transducer output by equation (8a) and δ as a function of y by equation (8b) when expanded in a Taylor series.

$$e_{DT} = Ay \tag{8a}$$

$$\delta = B'y + B''\frac{y^2}{2!} + B'''\frac{y^3}{3!} + \dots \tag{8b}$$

Higher than first order terms become negligible when the temperature gradient is linear. Substitution of this approximation of equation (8b) in equation (8a) yields equation (9).

$$e_{DT} = \frac{A}{B'}\delta. \tag{9}$$

Cross-multiplication and time-averaging of the total hot-wire signal (which includes e_m and the uncorrelated noise components e_n) with the displacement transducer signal leads to equation (10).

$$\frac{\overline{(e_m + e_n)e_{DT}}}{\overline{e_{DT}^2}} = \frac{\overline{e_m e_{DT}}}{\overline{e_{DT}^2}} = \frac{B'}{A}\bar{I}R_0\alpha_0\frac{\overline{\hat{T}_1\delta}}{\overline{\delta^2}}. \tag{10}$$

The calibration constant $(B'/A)\bar{I}R_0\alpha_0$ can be obtained by measuring the hot-wire signal under static conditions for a small but constant displacement from its equilibrium position. The wire is first held at a fixed displacement of $+y$ from its equilibrium position. At this point, the time averaged anemometer and displacement transducer signals E_{+y} and $E_{DT,+y}$ are obtained.

Similar readings are then obtained at the $-y$ position. By performing this calibration in the same temperature gradient as the dynamic tests, leads to equation (11).

$$\frac{E_{+y} - E_{-y}}{E_{DT,+y} - E_{DT,-y}} = \frac{\Delta E}{\Delta E_{DT}} = \frac{B'}{A}\bar{I}R_0\alpha_0\frac{(\Delta T_1)}{(\Delta T_E)}. \tag{11}$$

Combining equations (10) and (11) yields equation (12) where the terms on the LHS are readily measurable.

$$\frac{\overline{e_m e_{DT}}}{\overline{e_{DT}^2}}\left(\frac{\Delta E_{DT}}{\Delta E}\right) = \frac{\overline{\hat{T}_1\delta}}{\overline{\delta^2}}\left(\frac{\Delta T_E}{\Delta T_1}\right). \tag{12}$$

If a sinusoidal perturbation in wire position is used so that $\delta = |\delta|\sin\omega t$, then $T_1 = |T_1|\sin(\omega t - \phi)$, and if ΔT_E is scaled to equal $|\delta|$ with a consequent rescaling of ΔT_1, equation (12) reduces to equation (13) at any given frequency of perturbation.

$$\frac{\overline{e_m e_{DT}}}{\overline{e_{DT}^2}}\left(\frac{\Delta E_{DT}}{\Delta E}\right) = \frac{|T_1|}{\Delta T_1}\cos\phi. \tag{13}$$

In terms of equation (7), $|T_1| = |T_1(s)|$ and $\Delta T_1 = |T_1(0)|$.

It is seen, therefore, that the term of equation (13) containing the cross-product gives a direct measure of the amplitude response of the wire's transfer function multiplied by $\cos\phi$. Since the measurement of ϕ is quite difficult because of the large turbulent temperature fluctuation component, a comparison of measured results with $|(1+\tau_1 s)/(1+\tau_2 s)|\cos\phi$ rather than $|(1+\tau_1 s)/(1+\tau_2 s)|$ of equation (7) is preferred. A plot of this function is shown in Fig. 7 for $\tau_2 = 1$ and various τ_1.

This method has the advantage that calibration of the hot-wire is possible in the presence of extraneous but uncorrelated fluctuating signal components. A separate measure of the temperature gradient is not required. Use of a sinusoidal displacement has the additional advantage that any minor deviations from linearity of the temperature gradient will have a negligible effect on the result as the sinusoid has its probability density concentrated at the extremes of the wire displacement cycle. It also avoids the high accelerations associated with a square wave test signal which would be the ideal signal if the linearity error of the temperature profile is to be eliminated.

EXPERIMENTAL APPARATUS

All tests were performed at the outlet of a 40:1 contraction from a settling chamber designed to produce a low turbulence flow free of swirl. A spirally

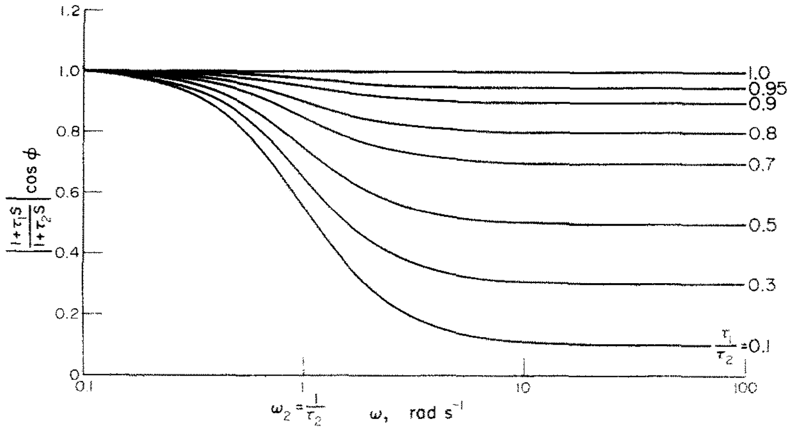

FIG. 7. Modified amplitude response of equation (7) for various τ_1/τ_2 with $\tau_2 = 1$.

wound heater was mounted in the settling chamber just upstream of the contraction to produce a core of air approximately 10°C above ambient temperature. The resultant temperature field contained regions of sufficient linearity of the temperature profile and flatness of the velocity profile for the tests to be possible.

The hot-wires were operated at constant current without compensation for their thermal inertia. Constructional details of the hot-wires and probe are shown in Fig. 8. The probe was designed for negligible flow distortion near the wire with the aid of hydrogen bubble flow visualization in a water tunnel. An electrodynamic shaker was used to perturb the hot-

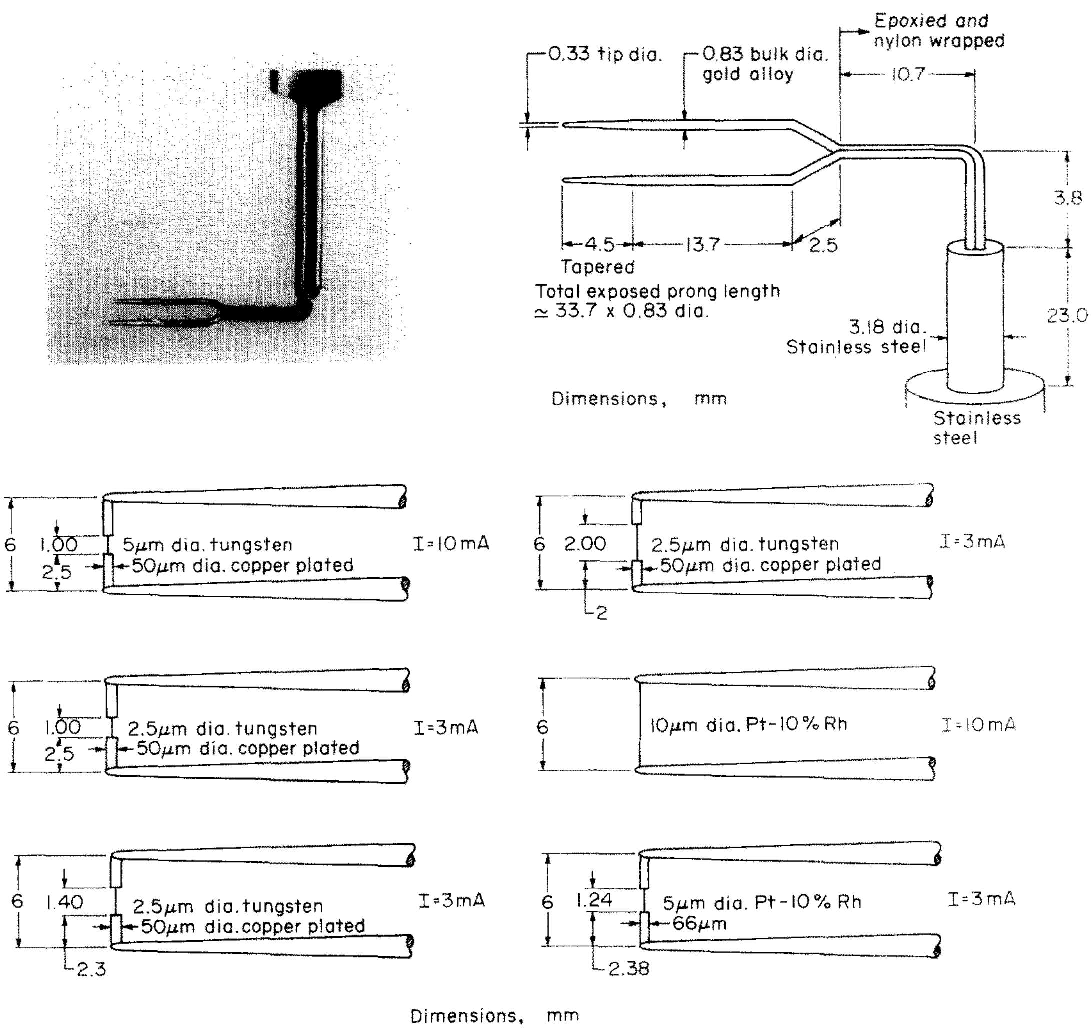

FIG. 8. Hot-wire and probe details.

wire probe, displacements being measured with a Hewlett–Packard 7DCDT displacement transducer. Checks for unwanted vibrational modes of the hot-wire probe were made with a stroboscope. Flow velocities at the hot-wire were measured with a pitot tube and Betz micromanometer.

Signal processing was carried out on an EAI 231R analog computer on-line. Test frequencies above 15 Hz could not be used for the following reasons, (i) the limited frequency response of the displacement transducer, (ii) to avoid the frequency range affected by the wire's thermal inertia, (iii) the limited capabilities of the electrodynamic shaker, and (iv) to avoid stray vibrational modes of the probe and hot-wire which severely reduce wire life.

Since the correlation method used is sensitive to amplitude and phase effects, considerable care was taken to avoid errors introduced by these in the signal processing equipment.

RESULTS AND DISCUSSION

Figures 9(a)–(d) show the results of dynamic tests for the wire-support combinations of Fig. 8 at three different velocities: points at only three frequencies are shown in Figs. 9(a)–(c) up to the maximum possible of 15 Hz with available equipment. The zero frequency point is not shown but is at $|T_1(s)/T_1(0)|\cos\phi = 1.0$. At lower frequencies than those shown, the statistical accuracy decreases as very long averaging times are required, up to 10 min, during which time the flow conditions tended to drift. Figure 9(d) shows typical results at the lower frequencies. Estimated experimental accuracy of $|T_1(s)/T_1(0)|\cos\phi$, and hence τ_1/τ_2, is 1–3% depending on frequency and flow velocity.

The data points shown together with that at $s = 0$ permit quite accurate curve fitting of the theoretically derived curves of Fig. 7. From this procedure, the value of ω_2 $(=1/\tau_2)$ for each test can be obtained and is shown on the abscissa of each plot together with the appropriate value of τ_1/τ_2.

Application of heat-transfer correlations for the estimation of the various resistances of the theoretical model of Fig. 3 shows that $R' \simeq R_{h3}$ to an accuracy of better than 5%. This is because the surface area of the supports available for convection heat transfer is large for the probe used. Similarly it can be shown that R_{h3} and R_{k3} are of similar order of magnitude. R_{k3} is independent of velocity but R' and R of equation 4(b) are dependent on it. τ_2 of equation (4d) can therefore be expected to decrease with increasing velocity. Figures 9(a)–(d) verify this trend.

It is of interest to assess the ratio of $R_{h3}/R_{k3} \simeq R'/R_{k3}$ to test the hypothesis stated at the outset that conduction heat transfer along the supports to a sink somewhere within the probe stem takes place. Taking as example the 5 μm tungsten wire at 5 m/s of Fig. 9(a) gives $\tau_1/\tau_2 = 0.73$. From current injection tests, Bremhorst *et al.* [4] found that for such a wire $R_{h1}/R_{c1} \simeq 1.35$. Substitution of this into equation (4c) for large R_{h2} together with equation (4e) yields $R = 0.73$ which with equation (4b) leads to $R'/R_{k3} = 2.7$. This value agrees well with one obtained by estimation using readily available heat-transfer data. The existence of R_{k3} with a value similar to but lower than that of R_{h3} also explains the relatively lower values of τ_2 obtained here, to those expected if τ_2 were calculated just on the basis of R_{h3}.

Perhaps the most significant result for practical anemometry is that for a 2.5 μm tungsten wire, even at the relatively high L/d ratio of 800, $|T_1(s)/T_1(0)|\cos\phi < 1.0$ at the lower velocities tested. Use of a low thermal conductivity wire material such as the platinum alloy, Pt 10% Rh, ($k = 30.1$ $Wm^{-1}\,°C^{-1}$ at 20°C) gives a considerable improvement as seen by comparison of the 5 μm tungsten ($k = 163$ $Wm^{-1}\,°C^{-1}$ at 20°C) wire of Fig. 9(a) with the 5 μm Pt 10% Rh of Fig. 9(d) at the same velocity and similar L/d ratio. Results between these two would be expected for platinum wires ($k = 71$ $Wm^{-1}\,°C^{-1}$ at 20°C). Tests with a 10 μm Pt 10% Rh wire with $L/d = 600$ resulted in $|T_1(s)/T_1(0)|\cos\phi = 1.0$ in the range of velocities and frequencies tested thus giving a clear indication of the high L/d ratio required even for a low conductivity material in order to achieve at least within present experimental accuracy, the ideal response of $|T_1(s)/T_1(0)|\cos\phi = 1.0$, that is, negligible end losses.

The model presented can now be used for the extrapolation of the test results. Equation (4e) can be rewritten as equation (14) for the case of large R_{h2}. In the lumped parameter model $R_{h1} \propto (hdL)^{-1}$

$$\frac{\tau_2}{\tau_1} - 1 = (1-R)\frac{R_{h1}}{R_{c1}} \tag{14}$$

and $R_{c1} \propto Lk_W^{-1}d^{-2}$. A reasonable approximation for the heat-transfer coefficient is $h \propto U^{0.33}d^{-0.67}$. Substitution in equation (14) for a given probe design leads to the relationship of equation (15).

$$\frac{\tau_2}{\tau_1} - 1 \propto \left[1 - \frac{1}{1+R_{k3}/R_{h3}}\right]\frac{k}{(dU)^{0.33}}\left(\frac{L}{d}\right)^{-2}. \tag{15}$$

From the dynamic test results, R_{h3}/R_{k3} was found to equal 2.7 at 5 m/s. Equation (15) for the velocity range tested may, therefore, be further approximated by neglecting the variation in R_{h3} hence giving the approximation of equation (16) which may be verified

$$\frac{\tau_2}{\tau_1} - 1 \propto \frac{k}{(dU)^{0.33}}\left(\frac{L}{d}\right)^{-2} \tag{16}$$

with the data of Table 1. Within the limitations of the lumped parameter model approximation for this distributed system and the experimental accuracy of 1–3% for τ_1/τ_2, quite good agreement is obtained between the experimental data and the predicted trends. A disadvantage of the dynamic parameter identification method is that for τ_1/τ_2 close to unity the result of equation (16) is very sensitive to small errors.

Predicted wire lengths required for the various wires tested are shown in Table 1 for $\tau_1/\tau_2 = 0.99$ and 0.95, the 1% and 5% accuracy levels when assuming a wire response independent of frequency. For tungsten wires very high L/d ratios, of the order of 1000 and more, are

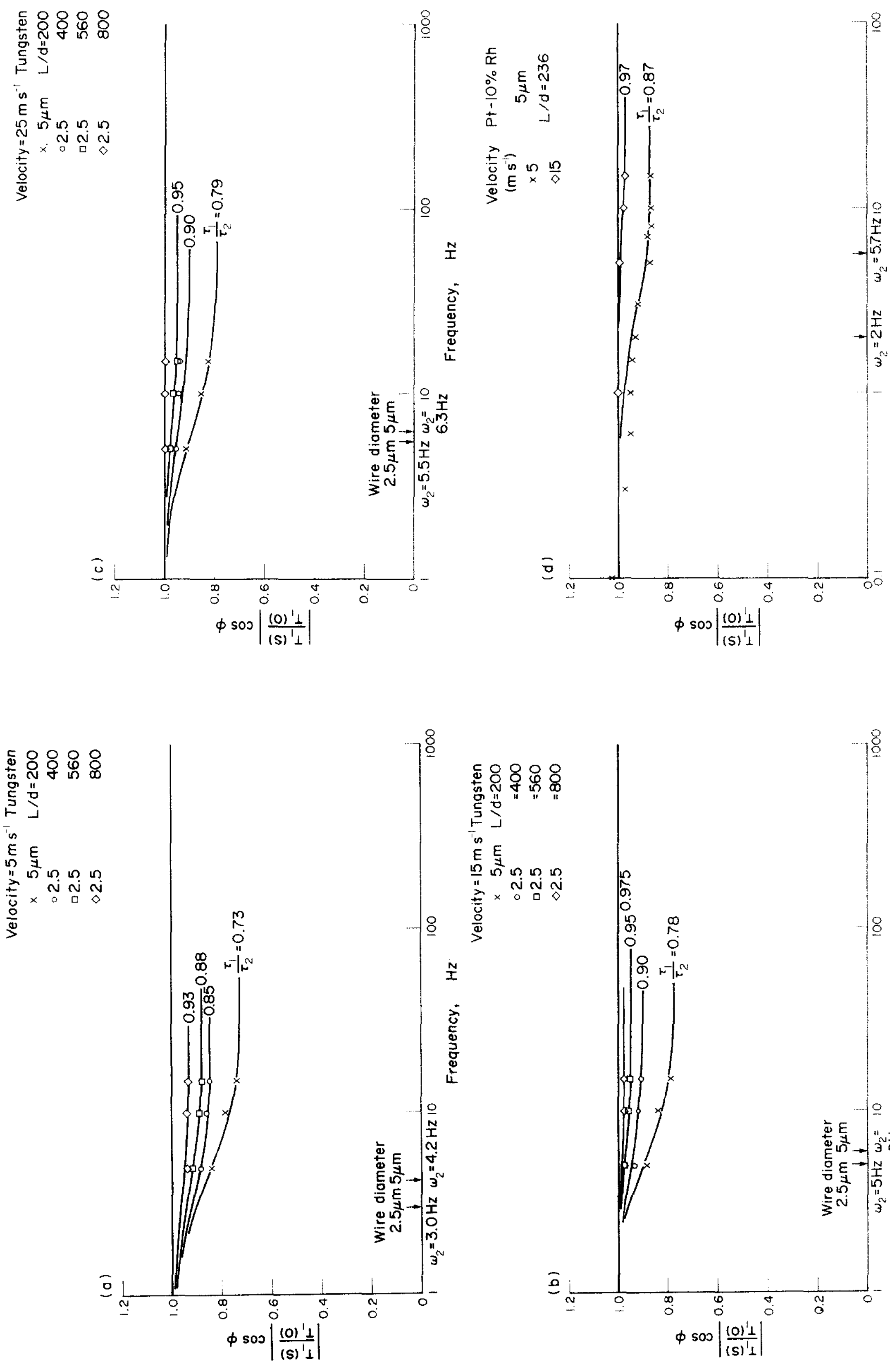

FIG. 9(a–d). Dynamic calibration test results.

Table 1. Extrapolation of test results at 5 m/s for reduced end conduction

Wire	τ_1/τ_2	L/d	L (mm)	Predicted L/d required for $\tau_1/\tau_2 = 0.99$	Predicted L/d required for $\tau_1/\tau_2 = 0.95$
2.5 μm tungsten	0.85	400	1.00	1670	750
	0.88	560	1.40	2080	920
	0.93	800	2.00	2180	980
5 μm tungsten*	0.73	200	1.00	1210	550
5 μm Pt 10% Rh	0.87	236	1.18	910	410
	0.83†	200			

* At 25 m/s this wire gives $\tau_1/\tau_2 = 0.79$ (by extrapolation).
† Extrapolated to $L/d = 200$ by use of equation (16).

required if a frequency independent sensitivity is to be achieved. The results clearly show the advantage to be gained by use of the Pt 10% Rh alloy as wire length is always to be as low as possible in order to improve spatial resolution.

In boundary-layer measurements, air velocities are often below 5 m/s which in the present experiments represented the worst case. Equation (16) indicates that τ_1/τ_2 is only a weak function of velocity. However, below 5 m/s, the term of equation (15) containing R_{h3} becomes significant. This leads to a more rapid drop of τ_1/τ_2 with decreasing velocity at which measurements with the available wind-tunnel were not possible due to significant buoyancy effects and their associated flow instabilities.

Equation (16) also indicates that for a given wire material and L/d ratio, τ_1/τ_2 and hence $|T_1(s)/T_1(0)| \cos\phi$ is almost independent of wire diameter. In fact, the larger diameter gives a slightly more favourable result. This is the reason that the extrapolated values of L/d required for a given accuracy of measurement (Table 1 for 2.5 and 5 μm tungsten wire) show that a slightly smaller L/d ratio can be used with the larger diameter wire. Accuracy of measurement is, therefore, strongly dependent on the length-to-diameter ratio and only weakly dependent on velocity and wire diameter.

Finally, the relevance of the above work to turbulent velocity measurements in non-isothermal flows requires consideration. In this case constant temperature anemometers are generally used which keep T_1 of Fig. 2 constant by varying I to meet the instantaneous cooling demand which can be produced by a change in R_{h1} or by δ. If δ is still assumed to be of sufficient spatial extent to affect all parts of the system uniformly, it is readily seen that end conduction will influence the response to δ since heat lost by convection is independent of the frequency of δ whereas that lost by conduction to the wire supports will be frequency dependent. Usually, such velocity measurements require correction for a substantial temperature component contained in the signal. The temperature sensitivity must, therefore, be known accurately. Frequently, 5 μm tungsten or platinum wires with L/d = 200–300 are used. The results of Fig. 9 are, therefore, applicable and indicate that considerable difficulty will be encountered if accurate correction for the temperature component of the signal is to be achieved.

CONCLUSIONS

Application of a novel cross-correlation, dynamic calibration technique has shown the significance of hot-wire end conduction losses for the measurement of stream temperature fluctuations under non-isothermal flow conditions.

A lumped parameter model of the wire and support system suitable for analysis of the wire's response under these conditions and consistent with the experimental results has been presented.

Results indicate that heat conduction along the wire supports is significant compared with the convection heat loss from the supports thus decreasing their time constant below the value applicable under conditions of convection heat loss alone. For low sub-sonic flows generally used in laboratory investigations and, particularly, in the wall region of boundary layers where velocities are always very low, results indicate that the Pt 10% Rh alloy wire is to be preferred. For such a wire of 5 μm diameter, an L/d ratio greater than 400 is considered to be necessary for measurement accuracies to within 5% in air flows with velocities down to 5 m/s if a wire response independent of frequency is assumed to apply. Since a long wire will lead to spatial resolution problems in turbulent flows, smaller diameters are to be preferred but the L/d ratio must still be similar as τ_1/τ_2 is only a weak function of wire diameter.

For measurements of turbulent velocities in non-isothermal flows where hot-wire signals generally contain a mixture of velocity and temperature contributions, wires with low conduction heat losses to the supports are to be preferred as uncertainties regarding the temperature component of the signal similar to those encountered with temperature sensitive wires will exist.

Throughout the above it was assumed that the stream temperature fluctuations are of large spatial extent. Application of the model presented, to smaller scale fluctuations is only of limited potential use as this is a distributed effect. However, if the above recommendations regarding wire selection are followed, uncertainties due to end losses are minimized and any extension of the model to small scale flows would not be of practical interest.

Acknowledgement—One of the authors (D. B. Gilmore) gratefully acknowledges receipt of a CSIRO Scholarship.

REFERENCES

1. J. P. Maye, Error due to thermal conduction between the sensing wire and its supports when measuring temperatures with a wire anemometer used as a resistance thermometer, DISA Information, No. 9, 22–26 (1970).
2. J. Højstrup, K. Rasmussen and S. E. Larsen, Dynamic calibration of temperature wires in still air, DISA Information, No. 20, 22–30 (1976).
3. J. P. Holman, *Heat Transfer*, Chapter 2, 4th edn. McGraw-Hill, New York (1976).
4. K. Bremhorst, L. Krebs and D. B. Gilmore, The frequency response of hot-wire anemometer sensors to heating current fluctuations. *Int. J. Heat Mass Transfer* **20** (4), 315–322 (1977).
5. K. Bremhorst and D. B. Gilmore, Comparison of dynamic and static hot-wire anemometer calibrations for velocity perturbation measurements. *J. Phys. E. Scient Instrum.* **9** (12), 1097–1100 (1976).

INFLUENCE DE LA CONDUCTION AUX EXTREMITES SUR LA SENSIBILITE DE L'ANEMOMETER A FIL CHAUD AUX FLUCTUATIONS DE TEMPERATURE D'UN ECOULEMENT

Résumé—On étudie l'influence de la conduction aux extrémités de l'anémomètre à fil chaud sur la sensibilité aux fluctuations de température, dans les conditions d'intensité constante et de faible surchauffe selon une nouvelle technique d'étalonnage avec corrélation. On présente un modèle thermique du système composé du fil et du support, compatible avec les données expérimentales. Les résultats montrent que pour les fils de tungstène et de platine généralement utilisés, on obtient une sensibilité qui dépend de la fréquence pour les fluctuations de température des écoulements d'air à vitesse subsonique. Pour des mesures à 5% dans des conditions d'écoulement non isotherme, on recommande l'utilisation d'un fil Pt—10% Rh avec un rapport longueur sur diamètre au moins égal à 400, indépendamment du diamètre, on a 800 pour un fil de tungstène. On discute aussi l'application des résultats aux mesures de vitesse avec des anémomètres à température constante.

EINFLUSS DER WÄRMELEITUNG ZU DEN ENDEN AUF DIE EMPFINDLICHKEIT EINES HITZDRAHT-ANEMOMETERS BEI TEMPERATURFLUKTUATIONEN IN DER STRÖMUNG

Zusammenfassung—Es wurde der Einfluß der Wärmeleitung zu den Enden auf die Empfindlichkeit eines Hitzdraht-Anemometers bei Temperaturfluktuationen in der Strömung untersucht. Das Anemometer wurde bei konstantem Strom und geringer Überhitzung betrieben, wobei eine neu entwickelte dynamische, gegenseitig ergänzende Eichmethode benutzt wurde. Ein einfaches Wärmeübertragungsmodell für den Draht und dessen Haltevorrichtung wird vorgestellt, welches mit den experimentellen Ergebnissen übereinstimmt. Die Ergebnisse zeigen, daß man mit den üblicherweise benutzten Wolfram- und Platindrähten in Luftströmungen bei Geschwindigkeiten weit unterhalb der Schallgeschwindigkeit eine frequenzabhängige Empfindlichkeit gegenüber Temperaturfluktuationen in der Strömung erhält. Für Messungen mit einer Genauigkeit von ungefähr 5% bei nicht-isothermen Strömungsbedingungen werden Drähte mit niedriger Wärmeleitfähigkeit wie Pt 10% Rh mit einem Länge zu Durchmesser-Verhältnis von mindestens 400 empfohlen, unabhängig vom Durchmesser, verglichen mit 800 für entsprechende Genauigkeit bei Wolframdrähten. Die Bedeutung der Ergebnisse für Geschwindigkeitsmessungen in nicht-isothermen Strömungen mit einem Hitzdraht-Anemometer von konstanter Temperatur wird ebenfalls diskutiert.

ВЛИЯНИЕ КОНЦЕВЫХ ЭФФЕКТОВ ТЕРМОАНЕМОМЕТРА НА ЧУВСТВИТЕЛЬНОСТЬ К ФЛУКТУАЦИЯМ ТЕМПЕРАТУРЫ ПОТОКА

Аннотация — С помощью недавно развитой динамической корреляционной методики калибровки было исследовано влияние концевых эффектов (утечек тепла через концы нити) анемометра постоянного тока, работающего при низких перегревах на чувствительность к флуктуациям температуры потока. Представлена кусочно-параметрическая модель теплообмена системы нить-державка, обеспечивающая согласование с экспериментальными данными. Результаты показывают, что в потоках воздуха при низких дозвуковых скоростях чувствительность обычно применяемых вольфрамовых и платиновых нитей к флуктуациям температуры потока зависит от частоты. Для измерений в неизотермических условиях с точностью порядка 5% рекомендуется использование нитей, изготовленных из материала с малой удельной теплопроводностью, такого как 10% сплав платина-рений, с отношением длины к диаметру не менее 400 (независимо от диаметра нити), в то время как для достижения такой же точности в случае вольфрамовой нити необходимо иметь это отношение порядка 800. Также обсуждается достоверность результатов измерений скорости потока термоанемометром постоянной температуры в неизотермических условиях.